An Uncertainty Exercise for Incompressible Flow

Magnus Östberg, Leif Broberg, Michal Orych, Björn Regnström

FLOWTECH International AB, Gothenburg, Sweden

SUMMARY

This paper describes a verification exercise of a RANS code with a k-ω Baseline turbulence model. First a verification of the code by the use of Manufactured solutions, where a 2D boundary layer look alike is used. The order of accuracy found is higher than 1 but lower than 2.

Then an uncertainty analysis of the manufactured case and a 2D backward facing step is performed. For the used grids the solver has a clear problem of predicting convergent values for local flow quantities, but behaves much better for integrated quantities.

Introduction

The purpose of this work is the verification exercise for the 2nd Workshop on CFD Uncertainty Analysis in Lisbon. This gives an opportunety to compare the overlap of the error bars from the uncertainty estimation for different codes.

The calculations in this paper are done with the Navier-Stokes solver Chapman.

Numerical method

To model the flow, the steady-state RANS equations together with Menters k-ω Baseline (BSL) model (Menter, 1993) is used. The solid wall boundary condition for ω is treated according to Hellsten and Laine (1997) which also allows for treatment of rough walls, but this feature was not used in the present investigation.

The equations are discretized with a finite volume method. For the convective fluxes the approximate Riemann solver of Roe is used (Roe, 1981) (Kaurinkoski and Hellsten, 1998) (Vierendeels et al, 1999), while for the diffusive fluxes central differences around the cell face centers are used. Flux-correction with a min-mod limiter is used to increase the accuracy to second order in regions of smooth flow.

ADI is used to solve the equations. The tri-diagonal systems that are solved contains the first-order Roe convective terms and the second order diffusive terms, while the second-order flux corrections are used as an explicit defect correction. Each element in the tri-diagonal matrix is a 6x6 matrix. For each sweep a local artificial time-step is calculated based on the CFL and von Neumann numbers in all directions except the implicit one.

If it were not for the source terms in the turbulence equations the above described discretization will guarantee that k and ω are kept positive. To maintain this in the presence of the source terms, the negative parts of the k-ω source terms are Newton-linearized and treated implicitly (Merci et.al. 2000). Strictly this does not guarantee positivity unless a time-step restriction is added, but in practice the artificial time-steps based on convection and diffusion are short enough that negative values of k and ω do not occur.

Boundary Conditions

Two layers of ghost cells are used around the boundaries. The variables in these cells are calculated at the same time as in the interior i.e. the values are updated within the ADI iteration.

For this exercise two different boundary conditions are used. The first one is Dirichlet boundary condition where a value of variable is specified on the boundary and extrapolated to the ghost cells with second order accuracy. The second is Neumann boundary condition where a normal flux at the boundary is specified and used to lineary extrapolate to the ghost cells.

Since this treatment of the ghost points give to low order for the diffusive terms a special stencil is used for the diffusion along boundaries.

Uncertainty Estimation Procedure

An uncertainty estimation proposed by Eça, Hoekstra and Toxopeus (2005) is used. The following options were used:

- Determine the observed order of accuracy, p, from the available data.
- For 0.95 $\leq$ p $<$ 2.05, U_ϕ is estimated with the Grid Convergence Index proposed and the standard deviation U_{fit} of the fit: $U_\phi = 1.25\delta_{RE} + U_{fit}$.
- For 0 $<$ p $<$ 0.95, the same error estimate is made but is then compared with the value Δ_M multiplied by a factor of safety of 1.25, so that U_ϕ is obtained from: $U_\phi = min(1.25\delta_{RE} + U_{fit}, 1.25\Delta_M)$.

- For p $\geq$ 2.05, $U_\phi = max(1.25\delta^*_{RE} + U_{fit}, 1.25\Delta_M)$, where δ^*_{RE} is also calculated in the least squares root sense with p = 2.
- If monotonic convergence is not observed, $U_\phi = 3\Delta_M$.

Test Case 1: Manufactured solutions

Grids

A set of 7 stretched grids is used. The coarsest grid, which has 24x24 cells and its first cell center at $y^+ = 1$, is shown in fig. 1. The finer grids have a relative refinement compared to the coarsest grid of 2, 4, 6, 8, 10 and 12 respectively.

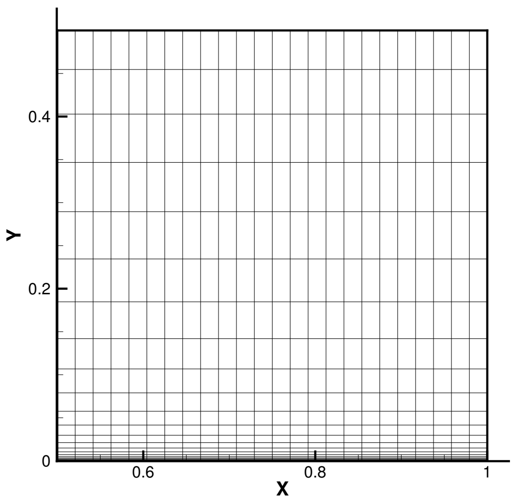

Figure 1: *Coarsest grid for Case 1*

Boundary Conditions

Dirichlet boundary conditions are used for velocity, k and ω for upper, lower and left boundary. For right boundary the flux of the same variables are specified. Pressure is specified at the upper, right and left boundary and a Neumann condition is used at the lower boundary. The turbulent viscosity is specified with Dirichlet conditions at all sides.

Results

The contours of u, v, Cp and ν_t for three of the grids are depicted in fig. 5-16. Grid convergence study show that the solver is more than first order accurate but less then second order accurate, see table 1 and fig. 17-20.

An uncertainty analysis is done in three different points for flow quantities u, v, Cp and ν_t. The extrapolated values and the corresponding uncertainty is presented in table 2. At point x=0.6 and y=0.001 no extrapolated value for Cp can be found.

Variable	**L1**	**L2**	**LInf**
u	1.85	1.87	2.04
v	1.52	1.47	1.51
Cp	1.59	1.51	1.04
k	1.54	1.47	1.40

Table 1: Order of accuracy

Also the friction resistance coefficient is tested with uncertainty analysis, see table 3.

Variable	**x=0.6,y=0.001**	**x=0.75,y=0.002**	**x=0.9,y=0.2**
u	0.00755	0.0120	0.791
U$_\phi$ for u	1.51e-05	5.12e-05	0.000283
v	6.35e-06	1.48e-05	0.0770
U$_\phi$ for v	1.72e-06	1.42e-05	0.000105
Cp	Divergence	0.0192	0.0161
U$_\phi$ for Cp	0.000214	0.000114	0.000296
ν_t	1.41e-10	9.25e-10	0.000323
U$_\phi$ for ν_t	6.58e-11	3.90e-10	1.57e-06

Table 2: Local flow quantities for Case 1

	Cf bottom	**U$_\phi$ for Cf bottom**
	3.16e-06	4.86e-08

Table 3: Cf at bottom wall for Case 1

Test Case 2: Backward Facing Step, Ercoftac Classic Database C-30

Grids

For case 2 the grids provided for the first workshop are used. The grids are grouped in three sets (SetA, SetB, SetC), see fig. 2-4. Each set contains seven grids with varying refinement.

Boundary Conditions

Dirichlet boundary conditions are used for u, v, k, ω and ν_t at the inflow and noslip boundaries. At the outflow the normal flux of u, v, k and ω is set to zero, while nu_t is extrapolated to the ghost cells. Pressure is set to zero at the outflow and Neumann condition with zero flux is used elsewhere.

Results

The contours for u, v, Cp and ν_t are depicted in fig. 33-68.

Again uncertainty analysis is done in three points for u, v, Cp and ν_t, see tables 4-6 and fig. 69-80. In about half

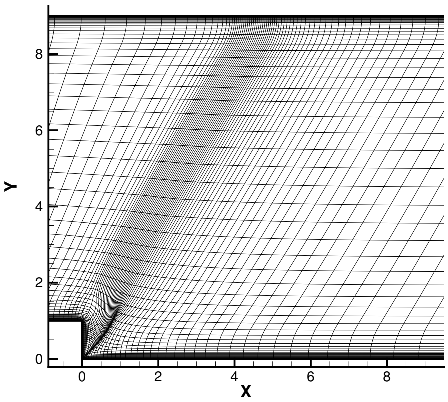

Figure 2: *SetA*

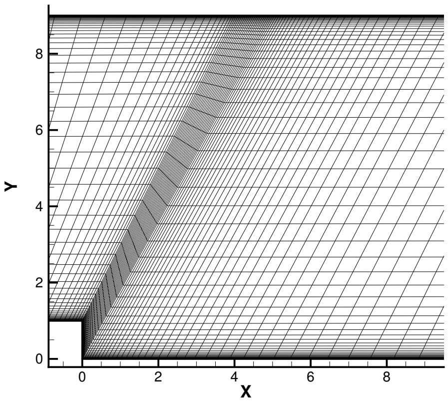

Figure 3: *SetB*

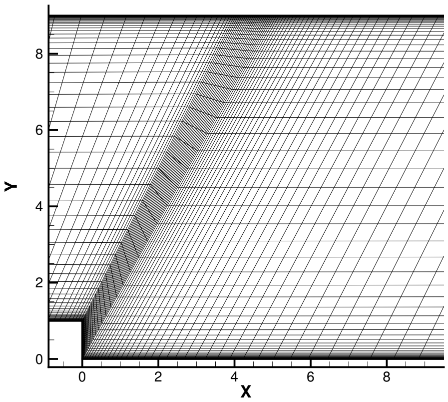

Figure 4: *SetC*

the cases the uncertainty analysis show divergence and for most of the cases the uncertainty is quite large.

In table 7 one can see extrapolated values and uncertainties for the re-attachment point, the friction resistance for the upper and lower walls and the pressure resistance for the lower wall. These values show much better convergence. Plotted values can also be seen in fig. 81-84.

Variable	**SetA**	**SetB**	**SetC**
u	Divergence	0.722	Divergence
$\mathbf{U}_\phi$ **for u**	0.0676	0.0165	0.209
v	Divergence	-0.418	Divergence
$\mathbf{U}_\phi$ **for v**	0.0101	0.011909	0.102
Cp	-0.0945	-0.104	Divergence
$\mathbf{U}_\phi$ **for Cp**	0.00671	0.0128	0.102
ν_t	Divergence	0.00151	Divergence
$\mathbf{U}_\phi$ **for** ν_t	0.000566	5.00e-05	0.00107

Table 4: x=0,y=1.1

Variable	**SetA**	**SetB**	**SetC**
u	Divergence	Divergence	Divergence
$\mathbf{U}_\phi$ **for u**	0.129	0.0987	0.0979
v	Divergence	0.0204	0.0402
$\mathbf{U}_\phi$ **for v**	0.00943	0.00437	0.00467
Cp	Divergence	Divergence	Divergence
$\mathbf{U}_\phi$ **for Cp**	0.0320	0.0147	0.00644
ν_t	Divergence	Divergence	Divergence
$\mathbf{U}_\phi$ **for** ν_t	0.00403	0.00310	0.00337

Table 5: x=1,y=0.1

Variable	**SetA**	**SetB**	**SetC**
u	Divergence	-0.420	-0.352
$\mathbf{U}_\phi$ **for u**	0.125	0.0474	0.0677
v	Divergence	-0.00856	-0.00891
$\mathbf{U}_\phi$ **for v**	0.000642	0.000418	0.000224
Cp	Divergence	Divergence	Divergence
$\mathbf{U}_\phi$ **for Cp**	0.0302	0.0231	0.0445
ν_t	Divergence	0.00596	0.00594
$\mathbf{U}_\phi$ **for** ν_t	0.000175	0.000123	0.000353

Table 6: x=4,y=0.1

Discussion

The values of omega are not included in the grid refinement investigation for obtaining the order of accuracy. The reason for that is the error norms for omega are clearly diverging. The explanation for that lies in omega being fixed to the correct values in the two cell layers closest to the wall. When refining the grid the region covered by these fixed cells decrease and in that region omega contain very large values and hence very large errors.

Variable	SetA	SetB	SetC
Re-attach	Divergence	6.27	7.10
U_ϕ for re-attach	1.22	0.397	0.669
Cf bottom	0.0315	0.0315	0.0315
U_ϕ for Cf bottom	0.00105	0.000958	0.00152
Cf top	0.0491	0.0491877	0.0491
U_ϕ for Cf top	0.00166	0.00163	0.00167
Cp bottom	0.0931	0.0960373	0.0909
U_ϕ for Cp	0.0195875	0.0207687	0.0131

Table 7: Re-attachment, friction resistance and pressure resistance

For the convergent cases of the uncertainty analysis for the backward facing step there is overlap for the error bars between grid sets in all cases but one. For v in point x=1 and y=0.1 the uncertainties would have to be more than twice as large in order to overlap between SetB and SetC.

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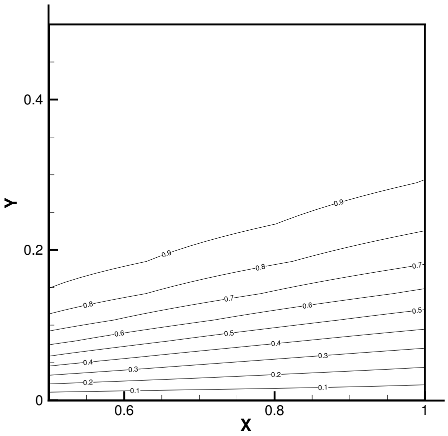

Figure 5: *u 25x25*

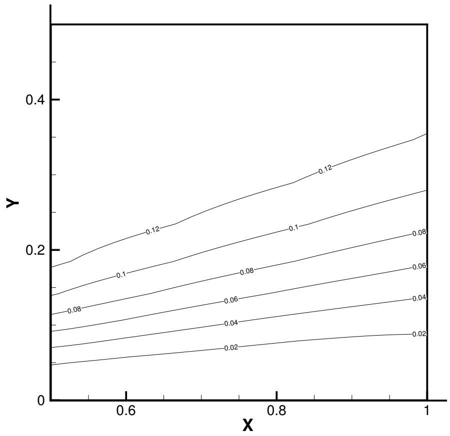

Figure 8: *v 25x25*

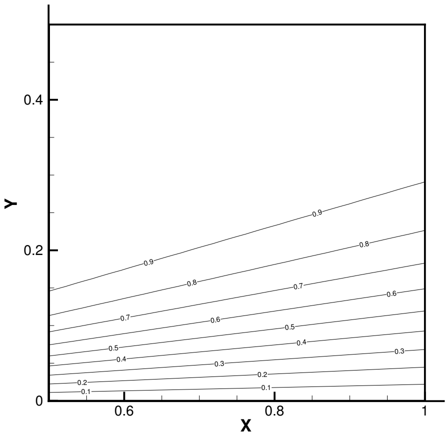

Figure 6: *u 97x97*

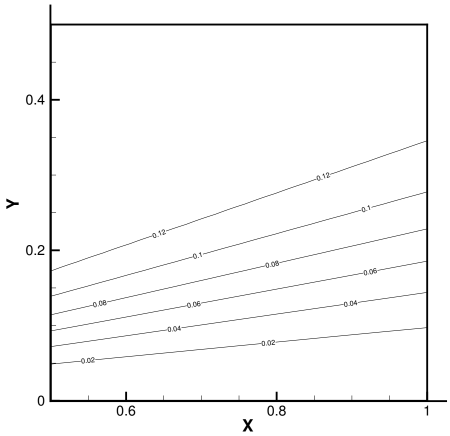

Figure 9: *v 97x97*

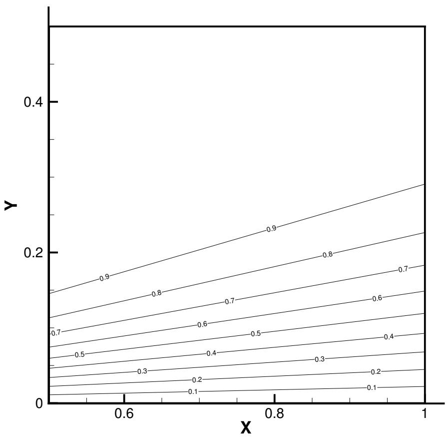

Figure 7: *u 289x289*

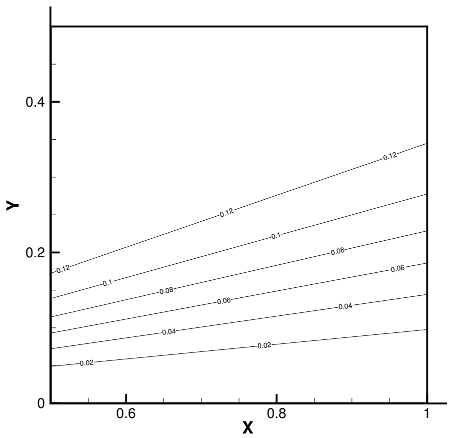

Figure 10: *v 289x289*

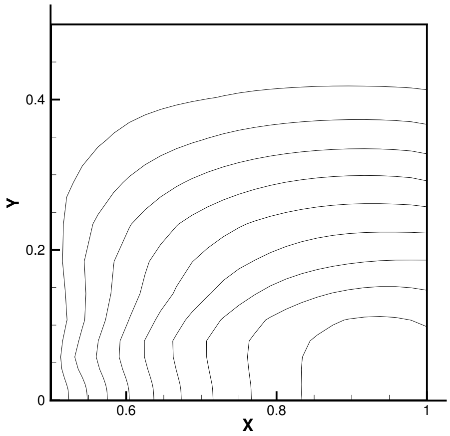

Figure 11: *Cp 25x25*

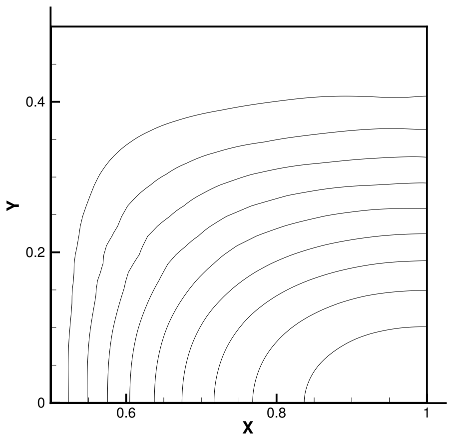

Figure 12: *Cp 97x97*

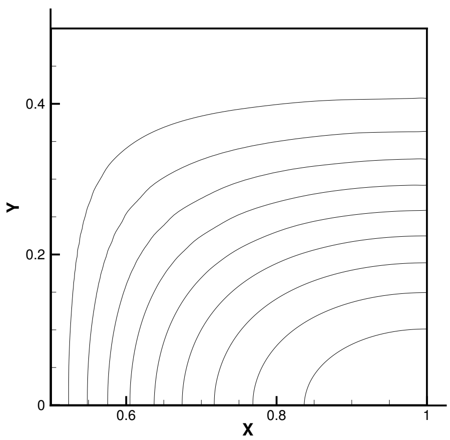

Figure 13: *Cp 289x289*

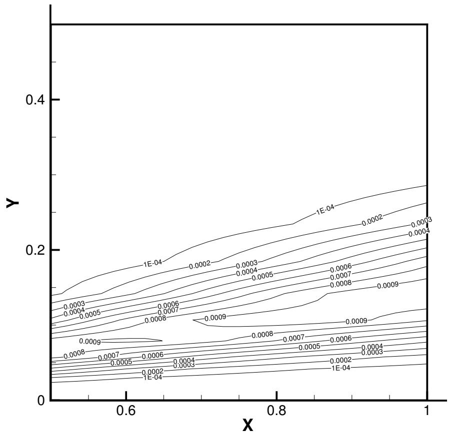

Figure 14: ν_t *25x25*

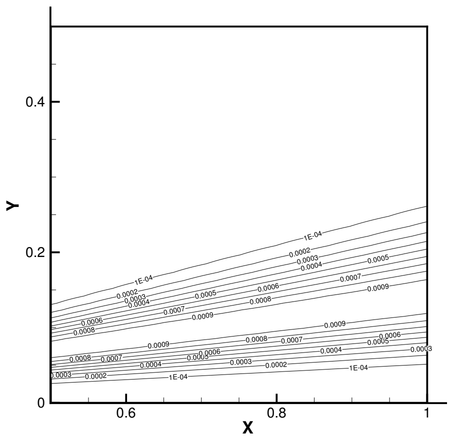

Figure 15: ν_t *97x97*

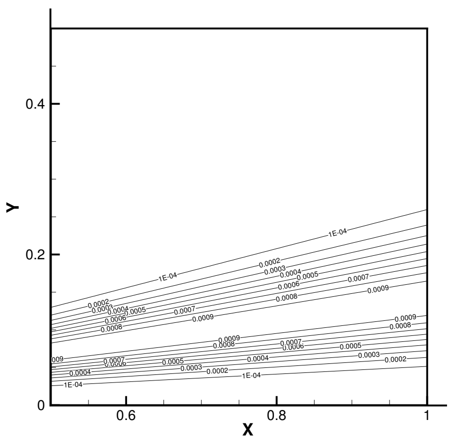

Figure 16: ν_t *289x289*

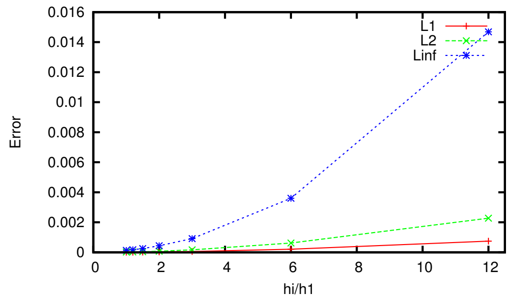

Figure 17: *Error plot for u*

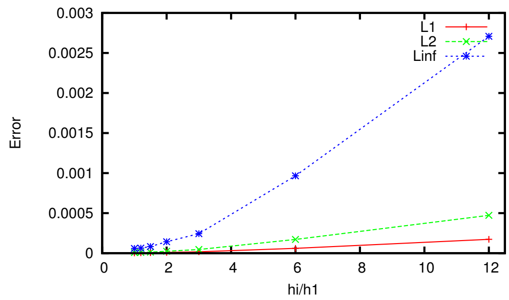

Figure 18: *Error plot for v*

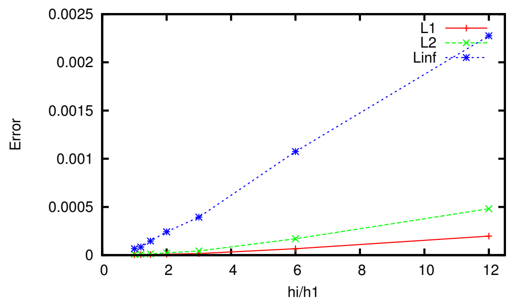

Figure 19: *Error plot for Cp*

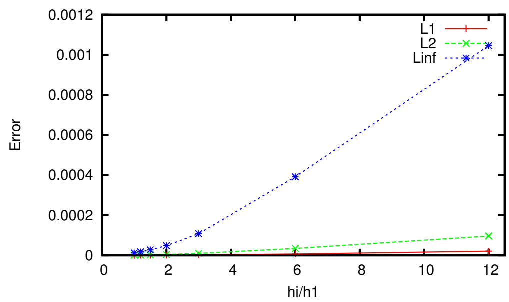

Figure 20: *Error plot for k*

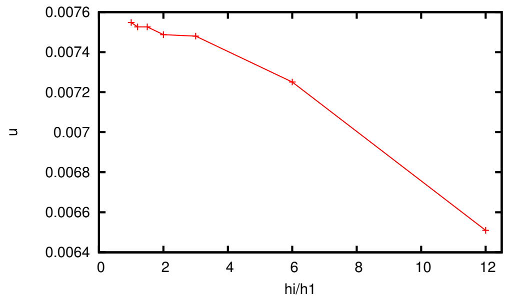

Figure 21: *x=0.6, y=0.001 Case 1 u*

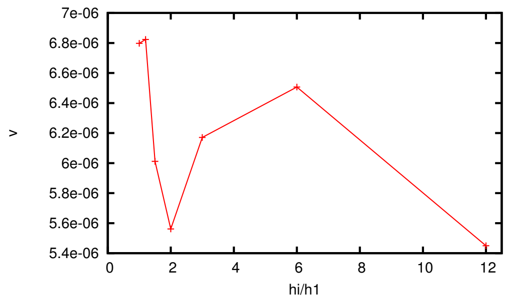

Figure 22: *x=0.6, y=0.001 Case 1 v*

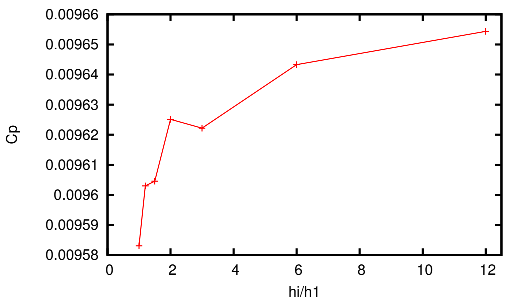

Figure 23: *x=0.6, y=0.001 Case 1 Cp*

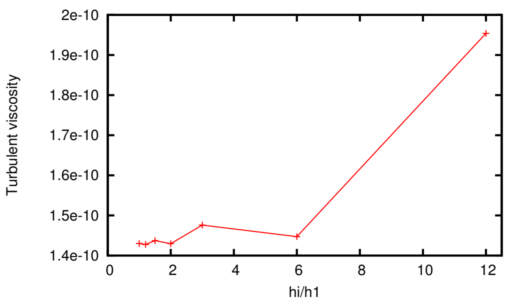

Figure 24: *x=0.6, y=0.001 Case 1* ν_t

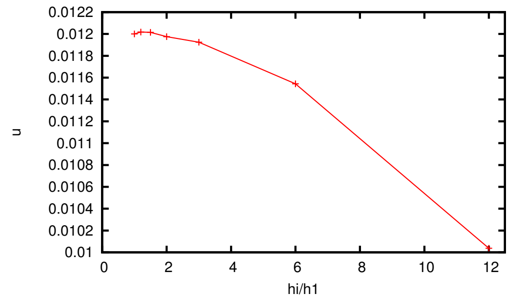

Figure 25: *x=0.75, y=0.002 Case 1 u*

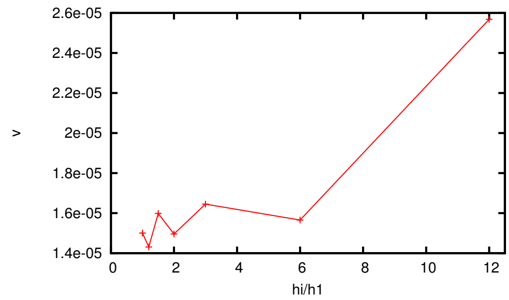

Figure 26: *x=0.75, y=0.002 Case 1 v*

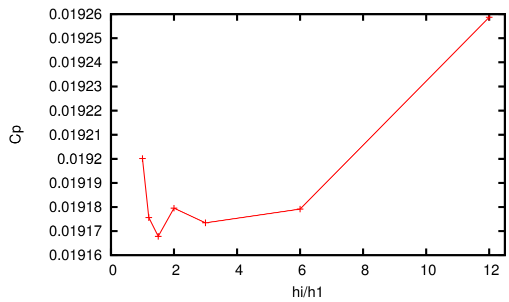

Figure 27: *x=0.75, y=0.002 Case 1 Cp*

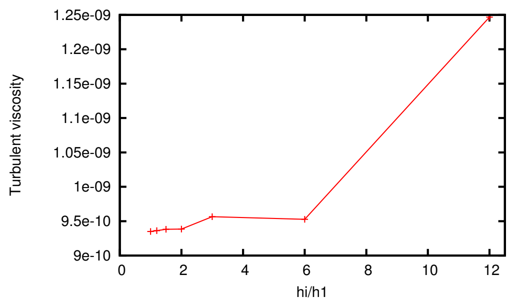

Figure 28: *x=0.75, y=0.002 Case 1* ν_t

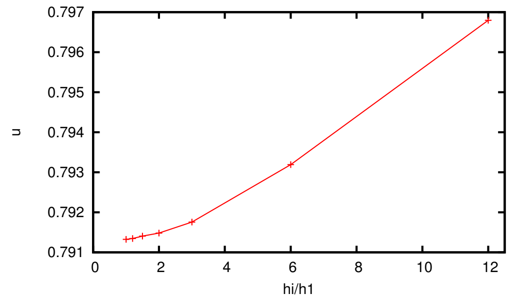

Figure 29: *x=0.9, y=0.2 Case 1 u*

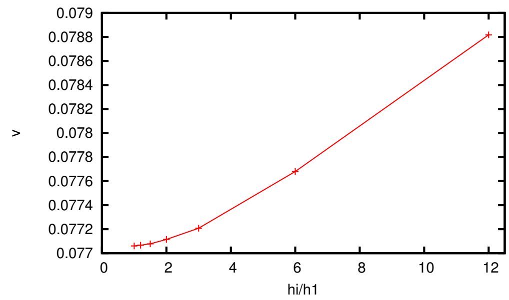

Figure 30: *x=0.9, y=0.2 Case 1 v*

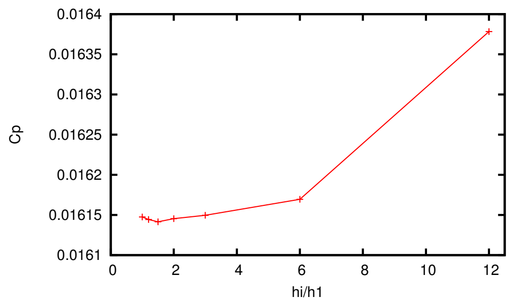

Figure 31: *x=0.9, y=0.2 Case 1 Cp*

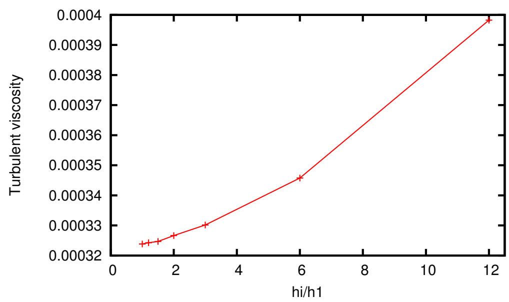

Figure 32: *x=0.9, y=0.2 Case 1* ν_t

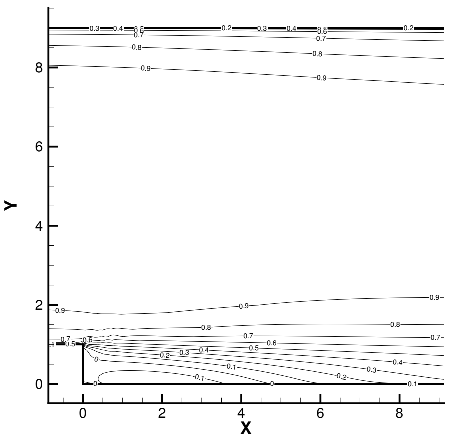

Figure 33: *u SetA 101x101*

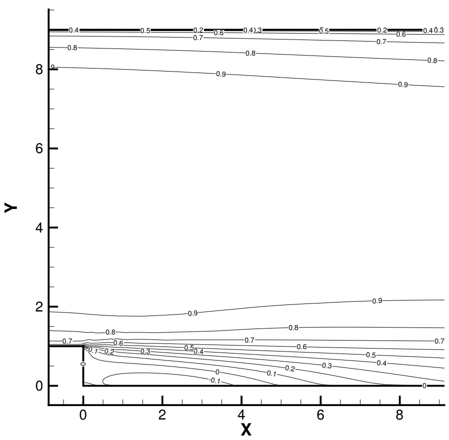

Figure 34: *u SetA 161x161*

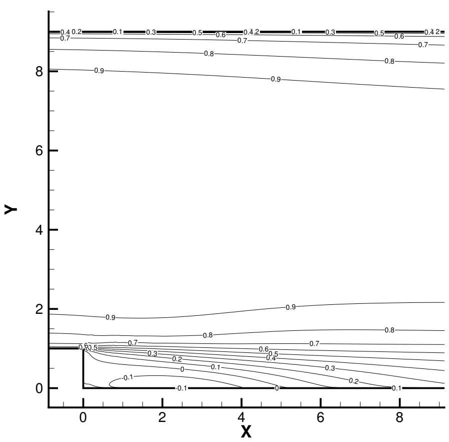

Figure 35: *u SetA 241x241*

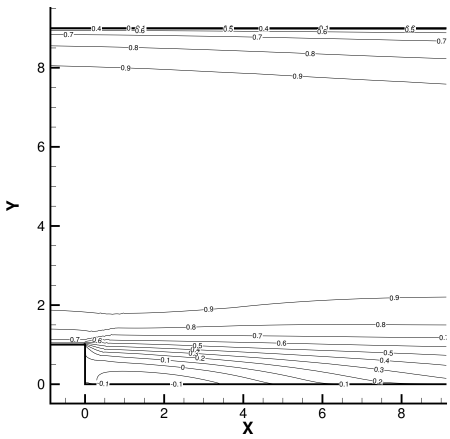

Figure 36: *u SetB 101x101*

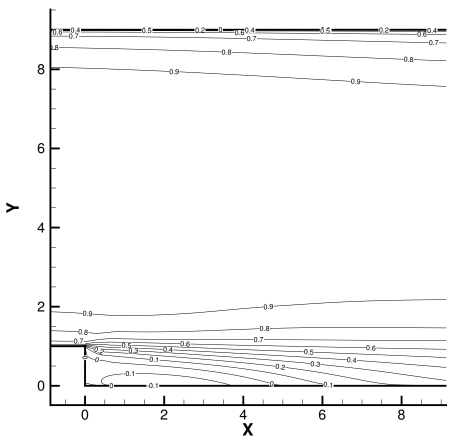

Figure 37: *u SetB 161x161*

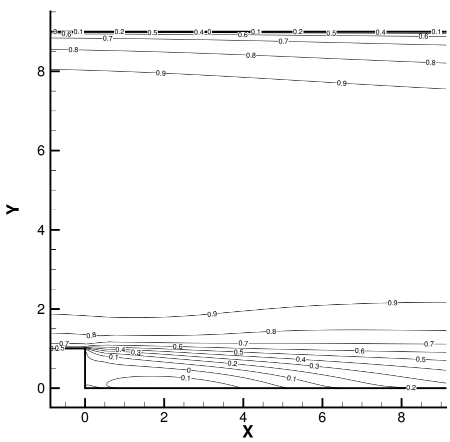

Figure 38: *u SetB 241x241*

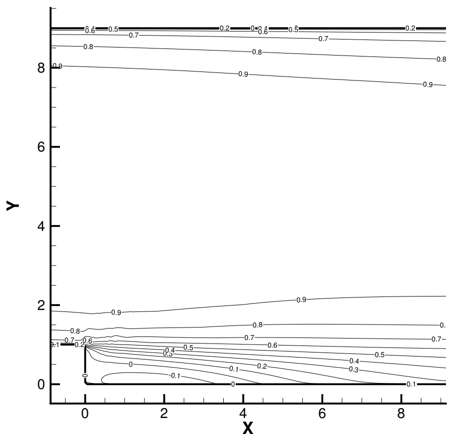

Figure 39: *u SetC 101x101*

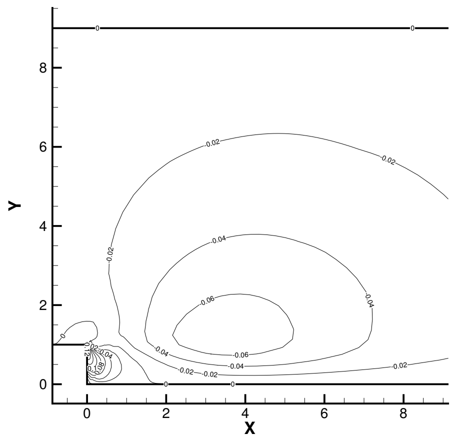

Figure 42: *v SetA 101x101*

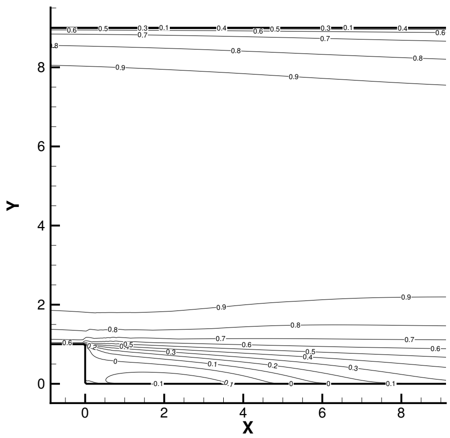

Figure 40: *u SetC 161x161*

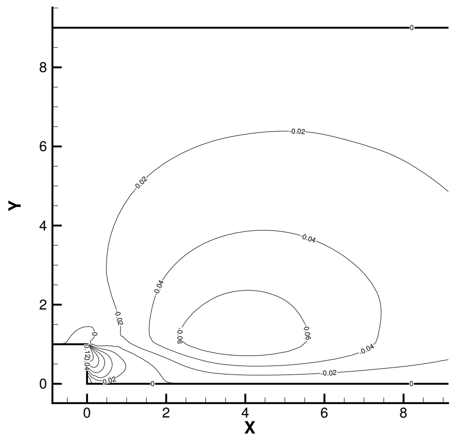

Figure 43: *v SetA 161x161*

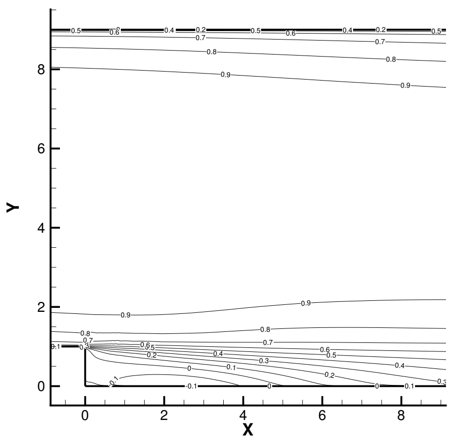

Figure 41: *u SetC 241x241*

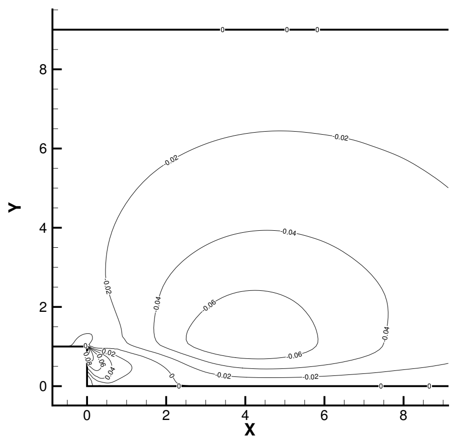

Figure 44: *v SetA 241x241*

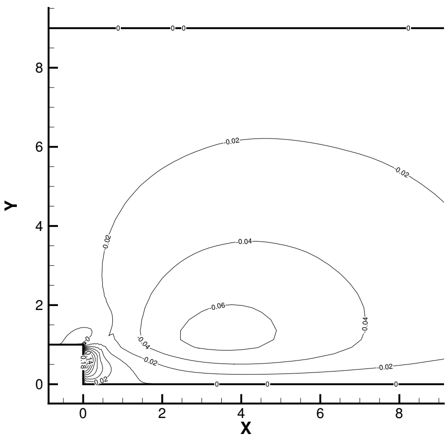

Figure 45: *v SetB 101x101*

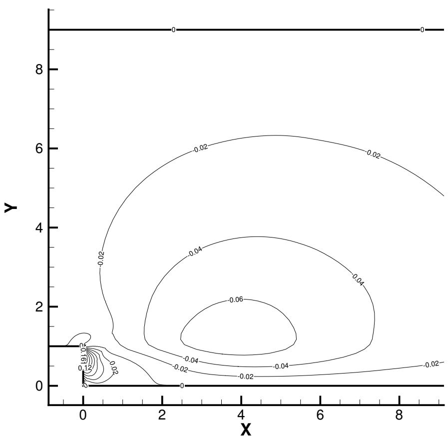

Figure 46: *v SetB 161x161*

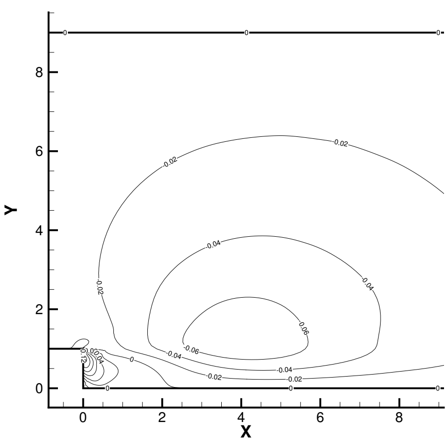

Figure 47: *v SetB 241x241*

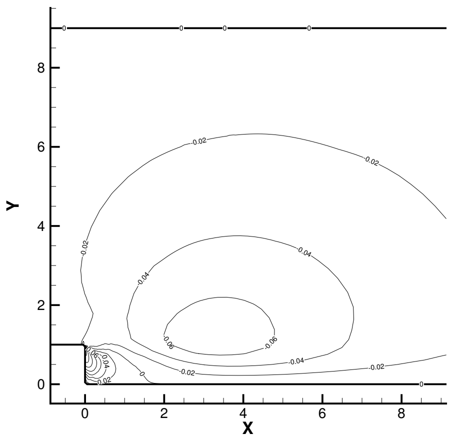

Figure 48: *v SetC 101x101*

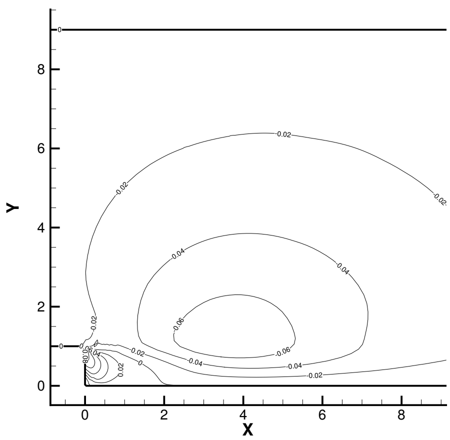

Figure 49: *v SetC 161x161*

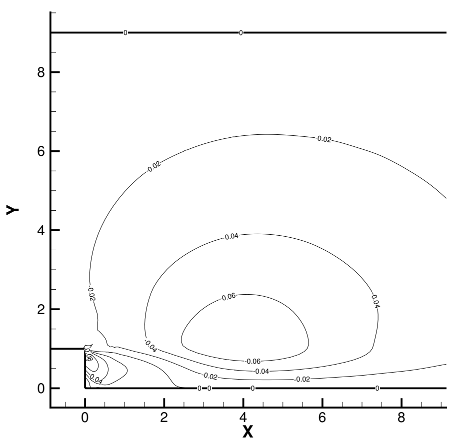

Figure 50: *v SetC 241x241*

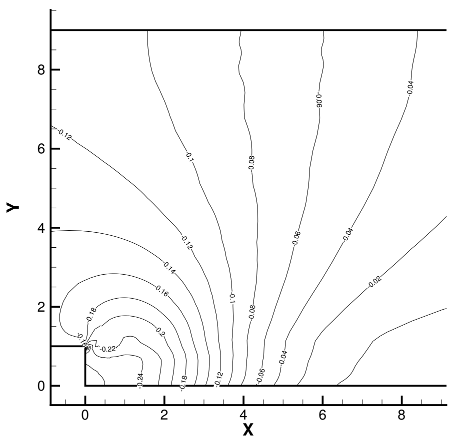

Figure 51: *Cp SetA 101x101*

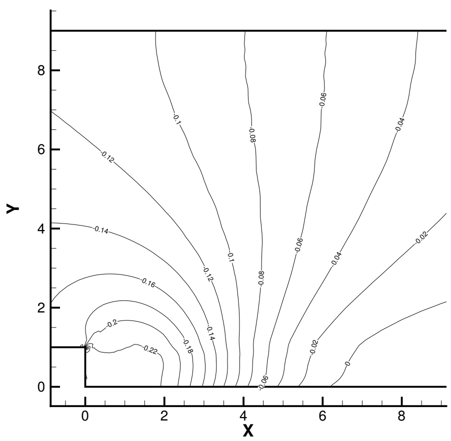

Figure 52: *Cp SetA 161x161*

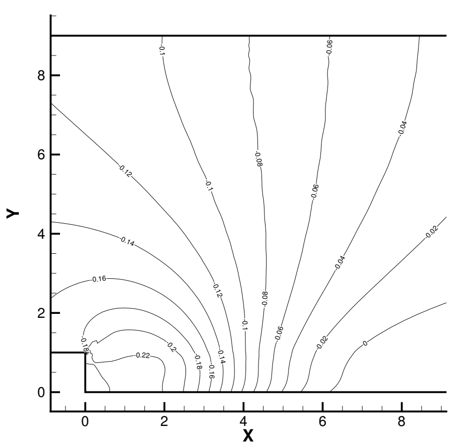

Figure 53: *Cp SetA 241x241*

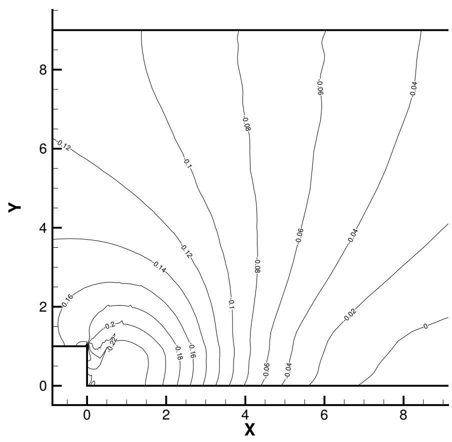

Figure 54: *Cp SetB 101x101*

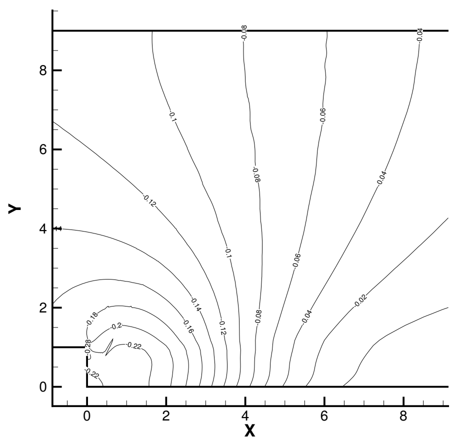

Figure 55: *Cp SetB 161x161*

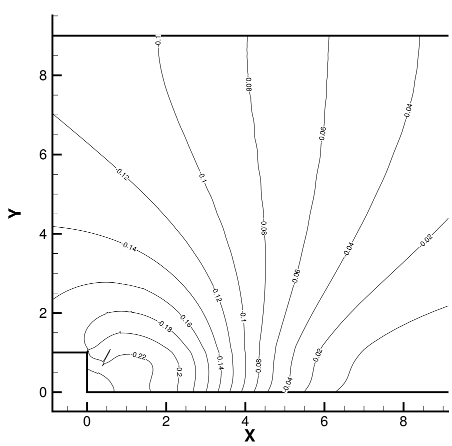

Figure 56: *Cp SetB 241x241*

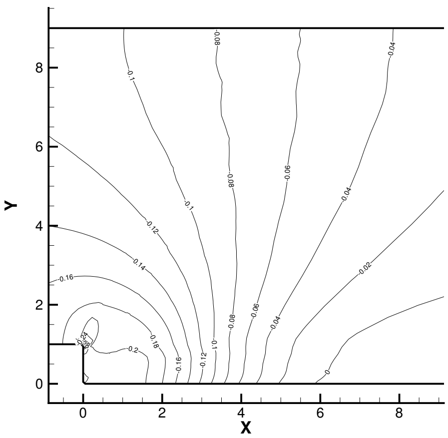

Figure 57: *Cp SetC 101x101*

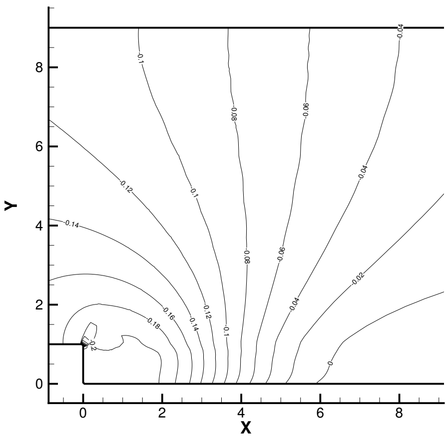

Figure 58: *Cp SetC 161x161*

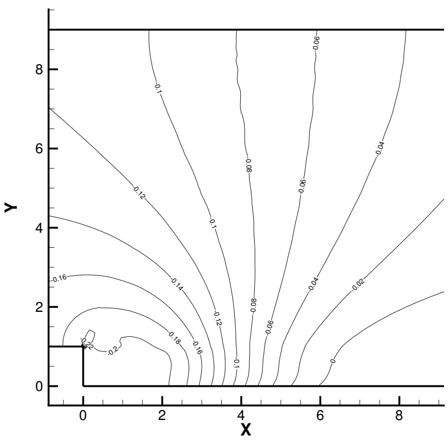

Figure 59: *Cp SetC 241x241*

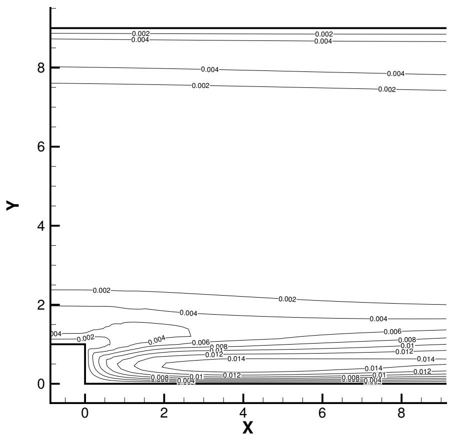

Figure 60: ν_t *SetA 101x101*

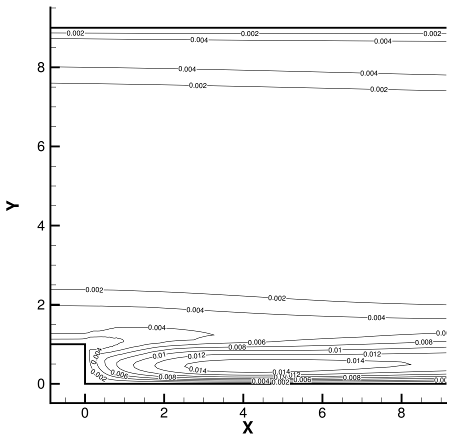

Figure 61: ν_t *SetA 161x161*

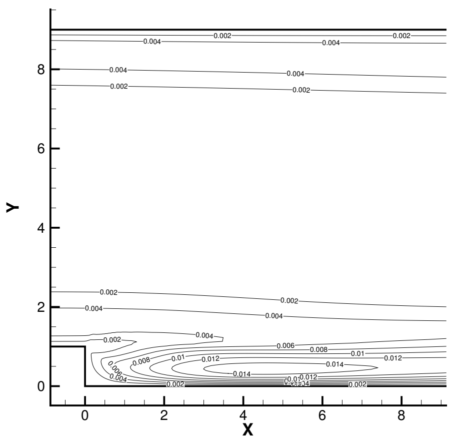

Figure 62: ν_t *SetA 241x241*

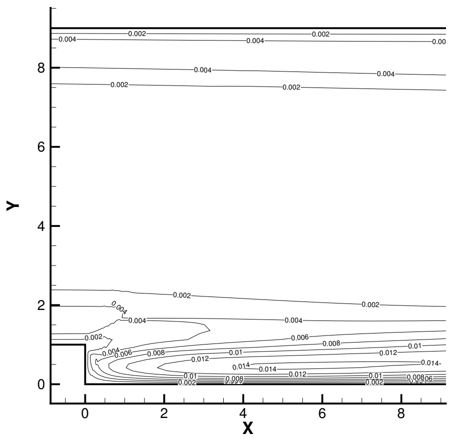

Figure 63: ν_t *SetB 101x101*

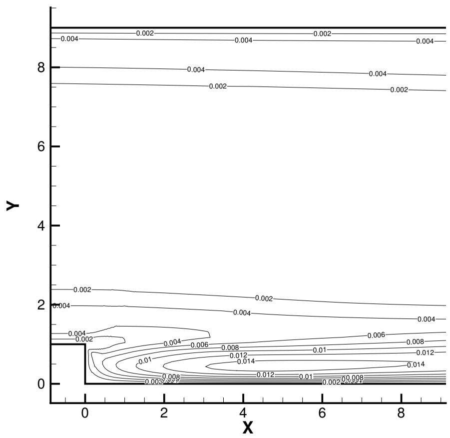

Figure 64: ν_t *SetB 161x161*

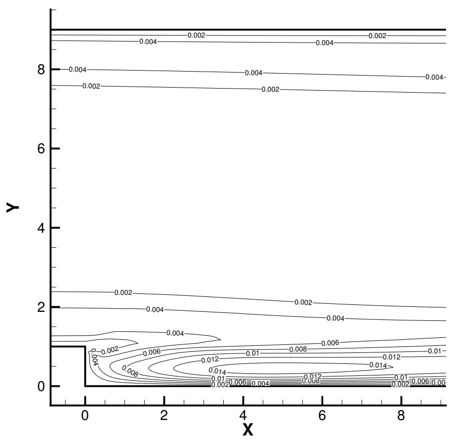

Figure 65: ν_t *SetB 241x241*

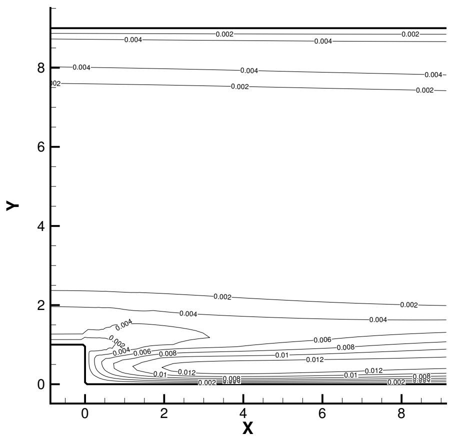

Figure 66: ν_t *SetC 101x101*

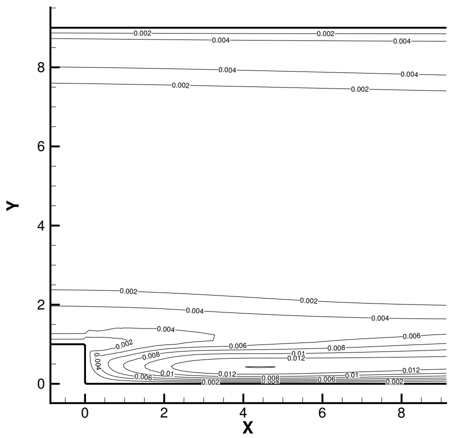

Figure 67: ν_t *SetC 161x161*

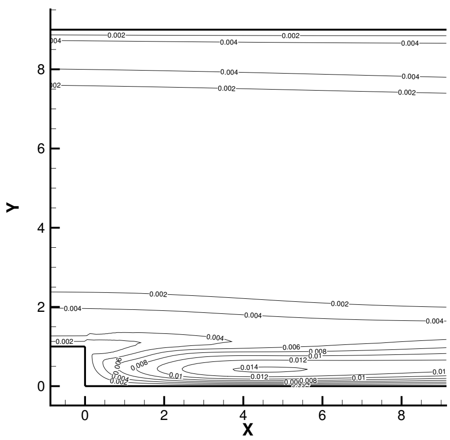

Figure 68: ν_t *SetC 241x241*

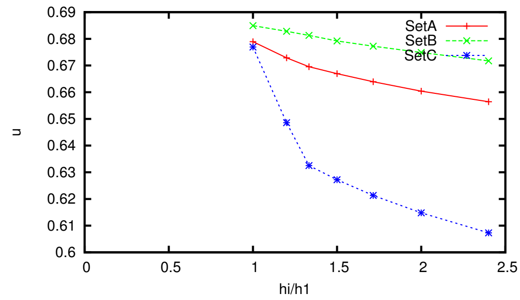

Figure 69: *u x=0, y=1.1*

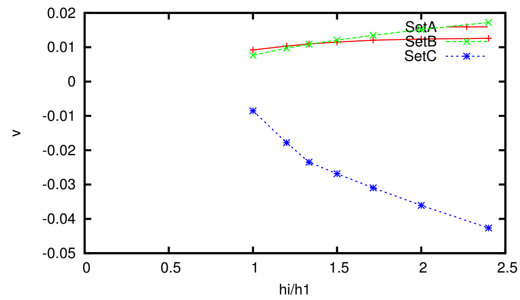

Figure 70: *u x=0, y=1.1*

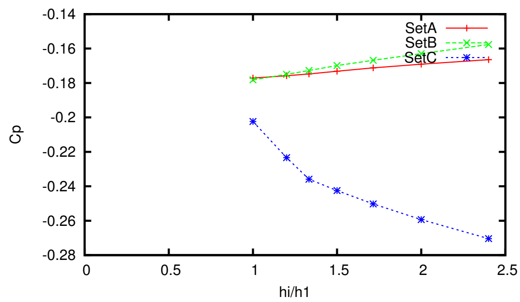

Figure 71: *u x=0, y=1.1*

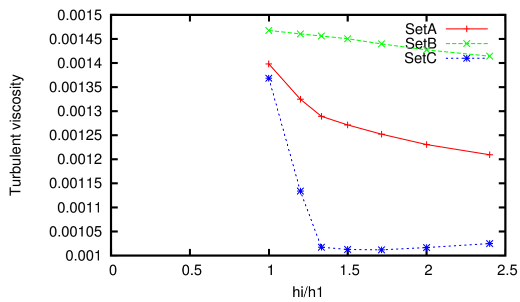

Figure 72: *u x=0, y=1.1*

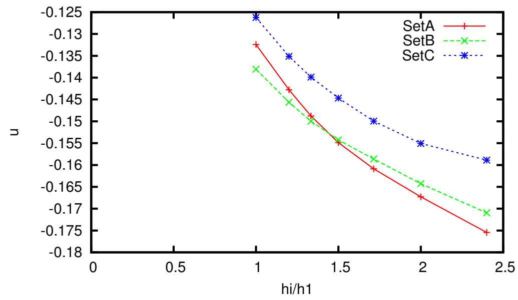

Figure 73: *u x=0, y=1.1*

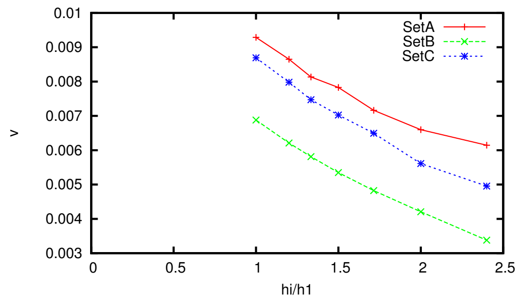

Figure 74: *u x=0, y=1.1*

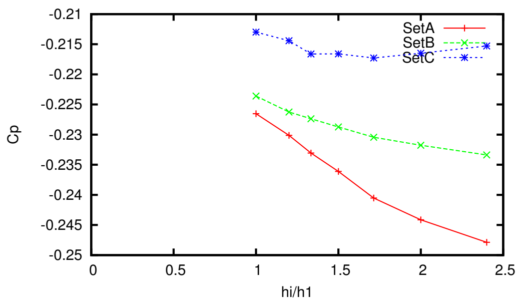

Figure 75: *u x=0, y=1.1*

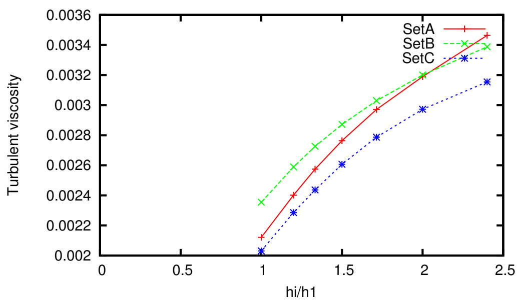

Figure 76: *u x=0, y=1.1*

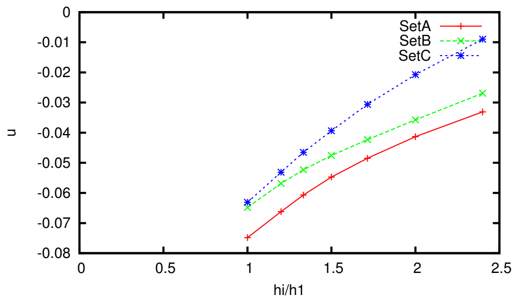

Figure 77: *u x=4, y=0.1*

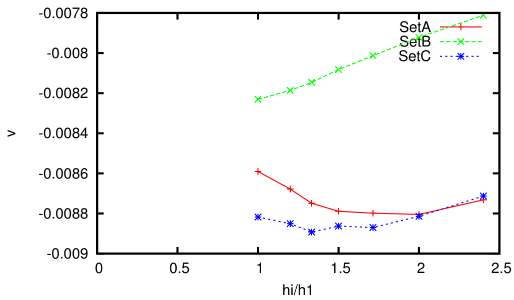

Figure 78: *u x=4, y=0.1*

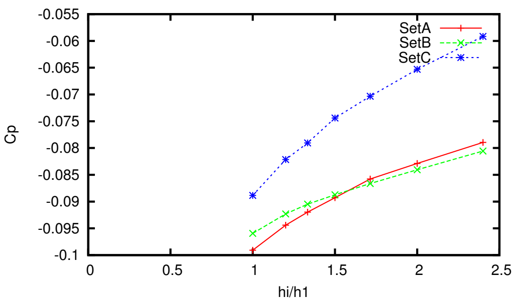

Figure 79: *u x=4, y=0.1*

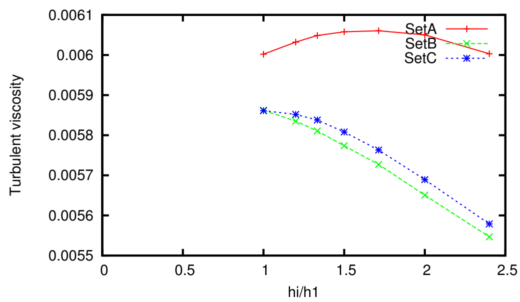

Figure 80: *u x=4, y=0.1*

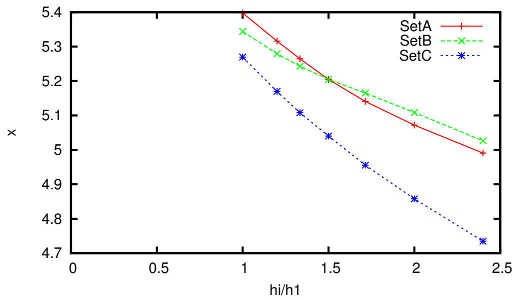

Figure 81: *Case 2 Reattachment point*

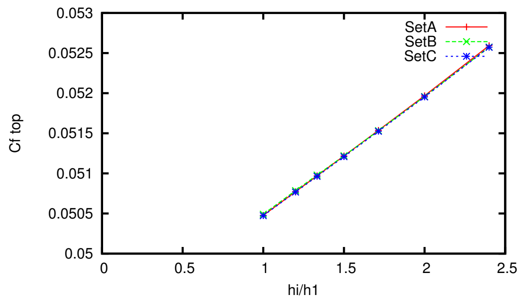

Figure 82: *Case2 Cf top*

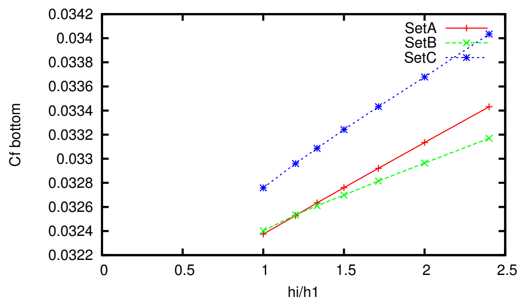

Figure 83: *Case 2 Cf bottom*

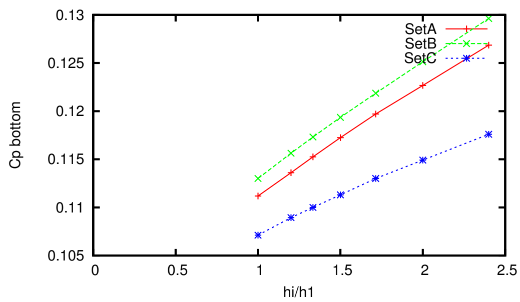

Figure 84: *Case 2 Cp bottom*