Summary of Workshop Discussion

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The discussion, if I remember well was that performing the tests on the manufactured solution for eddy viscosity models helped most participants achieve better assessment of the numerical uncertainties in the backward facing step.

The adaptive unstructured grid results for the k-epsilon model surprised many attendees. I confirmed several items about the unstructured grid error estimator and adaptation procedures:

- It can do with (n-1) grids what requires n grids with the GCI. With one grid the adaptive FEM yields a solution and an error estimate; the GCI requires two grids to get an error estimate.
- There are many ways to construct error estimators in the general Zhu-Zienkiewicz family, depending on
    1. what is being projected : a primary variables (u, v , t,…) or its gradient
    2. the nature of the least squares reconstruction : minimization of the integral of the square of a difference (continuous least-squares) or minimization of a summation of the square of a difference) (discrete, or super-convergent patch Least-Squares)
    3. the patch is node either node centered (Zhu-Zienkiewicz) or element centered (Wiberg)
    4. what norm is used to measure the error: L2, H1, point-wise
- We have produced results for global error estimates, friction resistance (i.e. drag, lift, moment coefficients on airfoils), and point error estimates. The a posteriori error estimator for the frictional resistance appears to be new. Global and friction resistance error estimates were obtained with the Zhou-Zienkiewicz approach, while point-wise estimates were obtained with the Wiberg approach. This is due to the fact the ZZ technique is based on projection of derivatives and estimates eerors in the derivatives only while the Wiberg approach projects the field variables themselves so that error estimates can be computed for the flow variables and their derivatives.
- Adaptive re-meshing is effective at generating grids that fall in the asymptotic range of the discrete approximation scheme.
- Using theoretical/formal rate of convergence in the grid design is at worst conservative; being optimistic on convergence rate is self correcting in that it refines the mesh at a slower pace.
- Since the workshop, we have pushed mesh adaptive simulations to extremely fine grids with up to 500,000 grid points! Results confirm the observations made in Lisbon. The global and friction resistance error estimators exhibit asymptotic exactness/

The technique can be extended to finite volume solvers. I implemented them for a cell centered finite volume method used in SECO-TP.

I think that much remains to be done to achieve a robust, reliable technique for error estimation, quantification of uncertainty, and error banding. Unstructured meshes offer great potential due to their geometric flexibility, and present great challenges. For example, the irregular topologies of such grids make it nearly impossible to apply the grid convergence index. Much work is needed to determine the reliability of the various reconstructions. Outstanding issues include application to highly stretched grids (turbulent boundary layers), dealing with transport of errors in convection dominated flows, coping with singularities caused sharp reentrant corners. Finally, unsteady flows represent another challenge due to the cross-coupling of time and space discretizations.

The two problems of this workshop should be part of the next workshop in 2008. This would be a perfect opportunity for participants to get a second crack at these problems. This is especially useful for those of us who where not satisfied with their results. In our case, we observed sub-optimal convergence rates that could be due to the combination of quadratic elements and stabilization schemes not well tailored to quadratic basis functions.